

Modelling the variation of bark thickness within and between European silver fir (*Abies alba* Mill.) trees in southwest Germany

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This study examined bark thickness variability of Silver fir (*Abies alba* Mill.) in southwest Germany over time and space by comparing a dataset of bark measurements from the 1970s with more recent assessments. Within-tree variability of bark thickness was analysed to estimate the required number of sampling locations per tree for defined accuracy levels. A range of models from the literature that predict bark thickness were compared for their predictive performance and Monte-Carlo simulations were used to estimate the effect of the number of sample trees and plots on the precision of the predictions. In addition, net log volume after bark subtraction was compared for logs of varying lengths to assess the influence of log assortments on calculated sales volume. Results show that several sampling locations are needed per tree and that at least five trees from at least 35 plots should be selected for measurements in the study region. For practical applications, diameter outside bark and breast height diameter are suggested as explanatory variables for models that predict double bark thickness. Additionally, relative tree height and age – and therefore growth rate – significantly improved predictions; however, environmental factors could not explain the variation between stands. Log lengths from 5 to 21 m only slightly influenced bark thickness equations that were fit on measurements at log midpoints. The findings highlight the need to consider bark thickness variability at different levels when developing bark thickness equations. In general, bark thickness was found to be smaller in more recent assessments and this indicates the need to regularly review existing bark equations for their validity.

Introduction

The diameter inside bark is a crucial measure in forestry that is used to calculate wood volume of logs and trees and to optimize bucking for defined inside-bark diameters. However, tree and log diameters are usually measured outside bark and bark thickness has to be estimated. Incorrect bark thickness or bark volume estimates could lead to inaccurate wood volume estimates in forest inventories, in increment studies or in the log trade (Marshall *et al.*, 2006). Furthermore, the interest in accurate bark thickness estimates is driven by the need to obtain not only accurate inside-bark diameters and volumes, but also accurate estimates of bark volume. The importance of accurate bark thickness estimates has increased with the shift in the commercial relevance of bark from an unwanted residue to a valuable fuel and feedstock for high-value biomaterial (Doruska *et al.*, 2009), such as bark tannin based foams (e.g. Pizzi, 2016). The estimation of available bark biomass is important to assess the potential of such technologies for generating possible additional income for the forestry sector. Additionally, bark volume estimates are needed for biomass estimates, which are becoming more important for quantifying carbon stocks (Temesgen *et al.*, 2015).

In this study, we use a common definition of bark that includes all tissues outside the vascular cambium, and comprises the secondary phloem up to the last formed periderm and the rhytidome, which comprises all layers of dead tissue outside the currently active periderm (Martin and Crist, 1970).

For many coniferous species, it has been shown that bark thickness can be described well by diameter outside bark at the sampling location and additional variables, as it often correlates with diameter at breast height, total tree height and height of the sampling location within the tree (e.g. Li and Weiskittel, 2011). Several external factors that correlate with the relative bark thickness of trees, i.e. the proportion of the outside-bark diameter that constitutes bark, have been reported. Most of these can be interpreted as a consequence of slower tree growth, which leads to a larger relative bark thickness. A correlation between a low site index or yield class of stands with a larger relative bark thickness was shown for Norway spruce (*Picea abies* (L.) Karst) (Hoffmann, 1958; Dimitrov, 1976; Schmidt-Vogt, 1986) and Silver fir (*Abies alba* Mill.) (Božić *et al.*, 2007). For Norway spruce (Laasasenaho *et al.*, 2005, Stängle *et al.*, 2017) and Silver fir (Božić *et al.*, 2007), tree age was shown to have an influence on the diameter-to-bark ratio. Sonmez *et al.* (2007) also reported a positive effect of tree age

on bark thickness for *Picea orientalis* (L. Link). Contrasting results were reported for the broadleaved *Nothofagus pumilio* (Poepp. & Endl.) Krasser by Cellini *et al.* (2012), where relative bark thickness was lower on low productivity sites. For *Pinus sylvestris* (L.), the latitude of forest stands – which mainly determines differences in temperature and length of the growing season – could help to explain variation in bark thickness (Wilhelmsson *et al.*, 2002). For *Pinus taeda* (L.) in the United States, there were no observed effects of silvicultural treatments, such as initial planting density and thinning, on whole-tree bark percentage. Only very intense treatments including fertilization and early-age weed control affected bark volume (Antony *et al.*, 2015). Besides growth conditions, genetics can also determine bark thickness. Provenance has been shown to influence the phenotypic development of bark thickness of *Pinus contorta* var. *latifolia* Engelm. (Persson and Downie, 1992) and *Pseudotsuga menziesii* (Mirb.) Franco (Kohnle *et al.*, 2012).

To develop accurate equations for predicting regional bark thickness, the intraspecific within-tree and between-tree variability of bark thickness have to be considered. Bark thickness variation within trees occurs at two levels. First, variation at the disc level around the circumference of the tree can be caused by a non-uniform bark thickness that is correlated with elliptical stem growth (e.g. Niklas, 1999) or a bark surface pattern with fissures, cracks or scales. The second level of within-tree variation occurs between sampling locations along the stem axis. To capture both sources of within-tree variability, several bark thickness measurements are usually taken at each of several sampling locations along the stem. For Norway spruce, variability within and between trees and between sites indicates the importance of multiple sampling points within each tree of a range of selected trees covering a range of sampling sites throughout the study region (Stängle *et al.*, 2016b).

Roundwood volume calculation in Germany is performed according to the Huber formula, which is based on a cylinder volume calculation using log length and the rounded diameter inside bark at log midpoint, which is located at exactly halfway along the log's length (Fonseca, 2005). In the study region, long logs (6–21 m) are mostly measured manually, whereas shorter logs from cut-to-length operations are usually measured at the mill site using electronic scanning devices. Measurement instructions for manual log measurement in Germany require the operator to take two diameter measurements at log midpoint (approximately perpendicularly) and round them to the lower full centimetre. The mean of these rounded values is also rounded down and species-specific fixed integer bark deduction values are subtracted to estimate the inside-bark diameter (Anonymous, 2015). Geospatial patterns of bark thickness within a study region could lead to skewed wood volume estimations if only one bark thickness equation and one set of fixed deduction values is used for the whole region. Additionally, relative bark thickness can vary along the stem and if the applied bark thickness function does not consider the position in the stem, volume calculations could be biased for different assortments. If long logs of up to 21 m length are produced, for example, most log midpoints would be positioned in the lower half of the stem, whereas midpoints of 5-m segments can basically be positioned all along the stem. The integer bark deduction values that are currently in use have been extracted from bark thickness equations that report double bark

thickness as a function of mid-diameters of logs of up to 26 m in length (Altherr *et al.*, 1978). However, different bark thickness equations could be useful for different assortments. Furthermore, bark thickness equations should be up-to-date as allometric relationships can change with changes in growth conditions. For Silver fir, growth conditions in the study area have changed in the last century with the average growth rate having increased since the 1990s (Kohnle *et al.*, 2014). Currently, bark thickness equations applied in Germany are based on measurements from the 1970s and probably overestimate bark volume as has already been shown for Norway spruce in southwest Germany (Stängle *et al.*, 2016a).

Using Silver fir also from southwest Germany, the objectives of this study were to:

- (1) estimate sample sizes required to capture bark thickness variation within trees and to examine the effect of number of sample plots and trees per plot on the precision of bark thickness equations;
- (2) compare and evaluate established model forms for estimating bark thickness and test how geospatial variation could be explained by environmental factors; and
- (3) estimate how log length and different datasets (measurements from the 1970s and 2010s) affect bark thickness equation coefficients for roundwood volume calculation.

Materials and methods

Data

Bark thickness data were obtained from two sources. Dataset 1, an extensive dataset with 3008 Silver fir trees, was collected by Altherr *et al.* (1978) in 82 clusters. Several clusters were sampled in the same stands, resulting in 50 sampled stands. Each stand was treated as one plot in the analysis. Dataset 2 consisted of newly acquired measurements on 217 trees in 27 plots. Measurements of dataset 2 were taken in the years 2015 and 2016. Dimensional details of all trees are listed in Table 1. Trees of both datasets were sampled throughout the 35 752 km² state of Baden-Württemberg in temperate forests with a broad range of site and stand conditions. Elevation of the plots from dataset 2 varied between 306 and 974 m above sea level (Figure 1). Mean annual air temperature varied between 5.8°C and 9.4°C and the annual precipitation was between 868 and 1944 mm. All stands were mixed-species stands with a range of ~20–80 per cent fir trees. Trees from a large range of merchantable diameter classes were selected from regularly harvested trees in selection cuts.

The sampling for both datasets was performed as described by Altherr *et al.* (1974; 1978): trees were felled with chainsaws, delimbed, and measured in the forest before any further log manipulation was performed. Measurement locations were at breast height (1.3 m above ground) and along the tree bole in 2 m increments up to a top diameter of ~10–15 cm. Both diameter and bark thickness were measured twice (approximately perpendicularly) at each location using a calliper and a Swedish bark gauge, respectively. Double bark thickness was calculated as the sum of the two gauge measurements. Additional measurements for the new data were tree age, and total tree height, which was measured on the felled tree. Tree height for dataset 1 and for trees with broken tips in dataset 2 was modelled with mixed B-spline regression describing tree taper using the package TapeR (Kublin and Breidenbach, 2015) in R (R Core Team, 2015). Two predefined bucking patterns were applied to produce virtual logs of different length from both datasets. The first bucking rule aimed at long logs. From each tree one virtual log

Table 1 Characteristics of the measured trees from dataset 1 and 2. Dataset 1 originates from Altherr *et al.* (1978).

	Dataset 1 (1970s)				Dataset 2 (2010s)			
	Mean	SD	Minimum	Maximum	Mean	SD	Minimum	Maximum
Age (years)					126.8	37.5	54	225
Tree height (m)					29.5	5.3	13.0	40.0
dbh (cm)	36.3	14.9	13.0	99.9	48.3	17.3	18.5	97.2
DBT _{1,3} (mm)	21.3	7.7	7.0	52.0	25.3	7.8	10.0	49.0

Note: dbh = diameter at breast height; DBT_{1,3} = double bark thickness at breast height; SD = standard deviation.

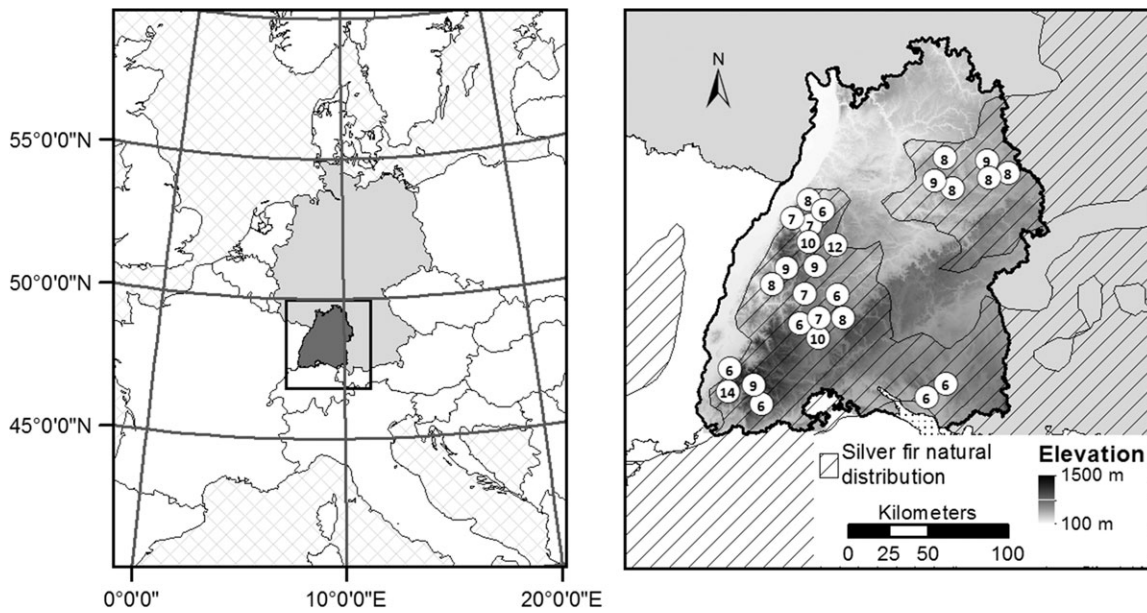


Figure 1 Distribution of sample plots in the study region and number of sampled trees per plot for dataset 2. The distribution map of Silver fir originates from (EUFORGEN, 2009).

between 6 and 21 m was generated. According to the second rule, several segments of 5 m length were generated from each tree. Due to convergence problems of the taper functions, only trees with a diameter at breast height (dbh) larger than 18 cm could be used for this analysis. For each log, diameter and bark thickness at midpoint and at the small end was calculated using taper equations based on inside-bark and outside-bark diameter.

To evaluate the effect of log length on equation coefficients and roundwood volume, the harvesting statistics of the State Forest Enterprise of Baden-Württemberg (ForstBW) were used. These data, hereafter referred to as dataset 3, included all manually measured logs of length longer than 6 m that were produced in the state forest of Baden-Württemberg in the period 2011–2015.

Bark thickness variability at different hierarchical levels

Bark thickness measurements within trees and within plots can be expected to be similar (e.g. Li and Weiskittel, 2011). The variance within trees determines the sampling design with the number of required sampling locations per tree and the number of measurements per location. If the variance between groups (such as trees and plots in our case) is large, additional information on group-level predictors could help explain this

variance when modelling bark thickness (Schielzeth and Nakagawa, 2013). To quantify the degree of variation that can be accounted for by different grouping levels, we fitted the established bark thickness prediction equation (1), which does not consider any predictor variables at the tree or plot level, but only the diameter outside bark at the measurement location.

$$DBT = \beta_0 + \beta_1 d_{ob} + \varepsilon \quad (1)$$

where DBT is the double bark thickness at any location along the stem (mm), d_{ob} is the diameter outside bark at that location (mm), ε is the residual of the model, β_0 , and β_1 , are the regression coefficients.

Thereafter, we calculated the intraclass correlation coefficient (ICC) as the ratio of between-group variances to total residual variance applying equation (2).

$$ICC(\%) = \frac{\hat{\sigma}_{\text{group}}^2}{\hat{\sigma}_{\text{total}}^2} * 100 \quad (2)$$

where ICC is the intraclass correlation coefficient, $\hat{\sigma}_{\text{total}}^2$ is the total residual variance, and $\hat{\sigma}_{\text{group}}^2$ is the variance within groups (trees or plots in our case).

Equation (1) was introduced by Loetsch *et al.* (1973) and used by Zacco (1974) in Sweden. Li and Weiskittel (2011, equation (7)) suggested

that it is useful when applied to spruce species. For model validation, 10-fold block cross-validation was performed. In order to do this, the plots were randomly assigned to one of 10 subsamples with each of them containing the same number of plots. In each of 10 runs, all measurements of one subsample were used as validation data and the measurements in the remaining nine subsamples were used as training data. Fit statistics include mean absolute error (MAE), root mean square error (RMSE) and mean percentage bias (PB). Results from all runs were averaged.

Calculating the required number of sampling locations per tree

For Norway spruce, Stängle *et al.* (2016b) suggested taking five and three measurements per sampling location for an allowable error of 15 and 20 per cent, respectively. The bark surface pattern of Silver fir is similar to that of Norway spruce, being smooth in earlier years and becoming scaly with flaking scales with increasing tree age (Freund and Grehn, 1951). Hence, the two species may be expected to have similar variability of bark thickness per cross-section, and the same number of measurements per sampling location seems to be adequate to reach similar accuracy.

The bark thickness variability along each tree bole was expressed by the variation of relative bark thickness within each tree. For each tree of dataset 1 (1970s) the number of required sampling locations per tree was calculated iteratively at predefined allowable errors of 15 and 20 per cent at a 95% confidence level using equation (3):

$$n = \left(\frac{CV \cdot t}{\text{allowable error (\%)}} \right)^2 \quad (3)$$

where n is the required sample size, $CV = SD(x)/\bar{x}$ (SD, standard deviation) and t refers to the 95 per cent-quantile of the two-tailed t distribution with $n-1$ degrees of freedom.

Monte-Carlo simulations to estimate the required number of sample trees and plots

To assess the influence of different sample sizes at the plot and regional levels on the predictive accuracy of equation (1), a Monte-Carlo simulation was performed on dataset 1 according to the method suggested in Stängle *et al.* (2016b): in each of 1000 runs, data were split in 35 plots for model fitting and 15 plots for model validation and equation (1) was fitted to the full training dataset and to thinned datasets representing different sample sizes at the tree and plot level. The tested sample sizes were 35, 20, 10 and 5 plots each combined with 20, 10, 5 and 1 trees. The plots and trees were drawn randomly with replacement from the training data as some plots contained fewer than 20 trees. The predictions from equations that were fit to the largest possible dataset, i.e. the full training data, were assumed to best capture the variability and were therefore considered the most optimal predictions. We used equivalence testing to determine the sample size required to have a high probability of fitting an equivalent equation as achieved with the full-data. To do so, in each of 1000 runs the most optimal predictions were compared with predictions from model fits achieved with thinned datasets using a robust two one-sided t -test (TOST) for equivalence (Robinson, 2016) with a region of equivalence of one millimetre ($P < 0.05$).

Evaluation of different bark thickness models

Six different models from the literature, which had different numbers of predictor variables at the disc and tree level, were compared for their fit quality and their predictive capacity on both datasets (1970s and

2010s). To account for the hierarchical data structure, mixed-effects modelling with random deviation of the intercept on the tree and plot level was applied, leading to equations (4–9):

$$DBT = \beta_0 + \beta_1 d_{ob} + b_i + b_{ij} + \varepsilon_{ijk} \quad (4)$$

$$DBT = \beta_0 + \beta_1 dbh_{ob} + \beta_2 d_{ob} + b_i + b_{ij} + \varepsilon_{ijk} \quad (5)$$

$$\log(DBT) = \beta_0 + \beta_1 d_{ob} + \beta_2 \log d_{ob} + \beta_3 \text{ind}_{dbh} + b_i + b_{ij} + \varepsilon_{ijk} \quad (6)$$

$$DBT = d_{ob} \left(\beta_1 + \beta_2 \frac{h}{H} + \beta_3 \left(\frac{h}{H} \right)^2 + \beta_4 H \right) + b_i + b_{ij} + \varepsilon_{ijk} \quad (7)$$

$$DBT = \beta_0 + \beta_1 d_{ob} + \beta_2 d_{ob}^2 + \beta_3 d_{ob}^3 + b_i + b_{ij} + \varepsilon_{ijk} \quad (8)$$

$$\log \left(\frac{DBT}{d_{ob}} \right) = \beta_0 + \beta_1 \left(1 - \frac{h}{H} \right)^{\beta_2} + \beta_3 \left(\frac{h}{H} \right)^{\beta_4 H} + \beta_5 dbh_{ob} + \beta_6 \frac{H}{dbh_{ob}} + b_i + b_{ij} + \varepsilon_{ijk} \quad (9)$$

where ind_{dbh} is an indicator for breast height (if the measurement was taken at breast height = 1, else = 0), h is the height of the sampling location above ground, H is the total tree height (m), b_i is the random effect for the i th plot, b_{ij} is the random effect for the j th tree in plot i , ε_{ijk} is the residual error for the k th measurement in the j th tree in plot i , β_{0-6} are the regression coefficients, and other variables are defined as above.

Equation (4) basically corresponds to equation (1) with the additional random effect terms. Equation (5) was introduced by Hannrup (2004) and is implemented in the harvester protocol Standard for Forest Data and Communication StanForD (Skogforsk, 2012). Equation (6) was found to be suitable for spruce by Wilhelmsson *et al.* (2002 equation (S5b)) and equation (7) was introduced by Cao and Pepper (1986 equation (4)) and recommended by Li and Weiskittel (2011 equation (4)). Model forms of equations (8) and (9) were described by Gordon (1983, equations (9 and 3)). The dependent variable of equations (6), (7) and (8) in the original publications was the diameter inside bark but was changed for this study so that the equations describe double bark thickness.

To account for positional autocorrelation within each tree, which was not eliminated by introducing the tree-level random effect, a first-order continuous autoregressive correlation structure (CAR1) was applied (Pinheiro and Bates, 2000), which has been used widely in forestry (Weiskittel *et al.*, 2011). This accounts for the similarity of bark thickness values within the tree along the stem, with closer positions being more similar to each other. A power variance function with d_{ob} as covariate was introduced to account for an observed larger residual spread for increasing diameters. The Bayesian information criterion (BIC) was computed to quantify an improvement of the model fit by introducing the above-described within-group correlation and heteroscedasticity structures.

Fit statistics include MAE, RMSE and PB. Predictions of the models (6) and (9) were corrected after logarithmic back-transformation using the correction factor $CF = \exp(SEE^2/2)$, where SEE is the standard error of the estimate (Baskerville, 1972; Sprugel, 1983). For the evaluation of the predictive capacity of the models (4)–(9), 10-fold block cross-validation was performed as described above for the validation of model (1). To obtain population-level predictions for the validation data, predictions were calculated without considering the nesting levels introduced in model fitting. Fit quality from the validation process was reported using the average of the MAE, the RMSE, and PB from the 10 runs. Mixed model fitting was performed using the `lme` function of the R package `nlme` (Pinheiro *et al.*, 2016).

To check if the two datasets from the 1970s and the 2010s led to significantly different parameter estimates, the model determined to

best describe both datasets was fitted to each dataset separately. Prediction bias was calculated in each case and residuals from both fits were compared with equivalence testing to see whether the model form could describe both datasets with similar accuracy. Subsequently, both fitted equations were applied to calculate double bark thickness predictions for both datasets and results were compared with equivalence testing.

Geospatial variation of bark thickness

To interpret the variation in bark thickness between stands or regions, we included spatial and environmental predictor variables in the modelling process. As exact stand location and tree age were only known for the newly collected data, this step could only be performed for dataset 2. We selected the predictors d_{ob} , dbh_{ob} , and h/H from equations (4–9), as they showed to significantly explain bark thickness variation. The predictor ind_{dbh} was ignored as it strongly correlates to h/H . To account for climatic effects, we included annual precipitation (mm) and mean annual air temperature (°C). Elevation of the sample plots (metres above sea level) and geographic coordinates (Gauss–Krueger projection, in m) were included to evaluate spatial effects that could be explained by climatic as well as by biotic or abiotic factors such as site index and soil materials, respectively. Additionally, we included tree age to account for the growth rate, which can be influenced by site quality and tree-individual factors, such as suppression by larger trees in a tree's early growth phase. Generalized additive mixed models (GAMMs) were chosen for this analysis and predictor variables were introduced as thin-plate-regression-splines using the `gamm` function of the R package `mgcv` (Wood, 2011). Geographic coordinates were introduced as tensor product splines to model a two-dimensional surface as suggested in Wood (2006). Thus, the applied equation (10) is similar to equation (4) plus relative height, age, the geographic surface and climatic variables:

$$\begin{aligned} DBT = & \beta + f_s(d_{ob}) + f_s(dbh_{ob}) + f_s\left(\frac{h}{H}\right) + f_s(\text{Age}) \\ & + f_{te}(\text{XCoord}, \text{YCoord}) + f_s(\text{Elev.}) + f_s(\text{Prec.}) \\ & + f_s(\text{Temp.}) + b_i + b_{ij} + \varepsilon_{ijk} \end{aligned} \quad (10)$$

where f_s stands for spline smooth functions, Elev. is the elevation, Prec. is the annual precipitation, Temp. is the mean annual temperature, $f_{te}(\text{XCoord}, \text{YCoord})$ is a tensor product spline of geographic coordinates, b_i is the random effect for the i th plot, b_{ij} is the random effect for the j th tree in plot i , ε_{ijk} is the residual error for the k th measurement in the j th tree in plot i , and other variables are defined as above.

Fit statistics of sub-models of equation (10) were compared after dropping single explanatory variables to test for the significance of each model parameter using the `dredge` function of the R package `MuMIn` (Barton, 2016). BIC was used to rank the fits achieved by this procedure.

Bark thickness equations for roundwood volume determination

To test the effect of different log lengths on the accuracy of bark thickness equations, taper curves of trees from dataset 2 were used to create a set of virtual logs of different length according to the above-described bucking rules (long logs and 5-m segments). From those logs, rounded mid-diameters and small end diameter were extracted as sets of reference diameters, on which bark thickness equations were fit. Additionally, the diameter and bark measurements from all sampling locations (every 2 m in each tree) were used as a control set of reference diameters. Equations (1) and (11) were fit on the different sets of reference diameters and the better fit for each set was selected using BIC. Both

models only have d_{ob} as the predictor variable; however equation (11) also includes it as a quadratic term:

$$DBT = \beta_0 + \beta_1 d_{ob} + \beta_2 d_{ob}^2 + \varepsilon \quad (11)$$

Depending on the species, one of equations (1) and (11) is currently used to estimate bark thickness at log midpoints of manually measured logs in Germany. For Silver fir, the currently used equation corresponds to equation (11) with a negative coefficient for the quadratic term leading to a flattening of the equation curve at larger diameters (Altherr et al., 1978).

After evaluating the effect of log length, the effect of different fitting datasets on the developed bark thickness equations was assessed. Therefore, equations (1) and (11) were fitted on mid diameters of virtual long logs from each dataset, and again, the better fit was chosen using BIC. From the developed bark thickness equations, integer bark deduction values of 1–4 cm were assigned to corresponding diameter classes. At the class midpoints 15, 25 and 35 mm double bark thickness, the class boundaries were set for deduction values of 2, 3 and 4 cm.

The economic implication of updated bark deduction values was evaluated by comparing the already established bark deduction values, which correspond to the calculated values from dataset 1 (Anonymous, 2015), and the newly calculated values from dataset 2. In order to do so, net log volume after bark subtraction was calculated using outside-bark diameter measurements of the logs from dataset 3.

Results

Sampling design and sample size

With increasing total tree height, more sampling locations within each tree were required to describe the variance of relative bark thickness (Figure 2). The calculated average number of sampling locations that was required for an allowable error of 15 per cent was 6.5 ± 3.3 (mean \pm SD). A 20 per cent allowable error resulted in an average of 4.2 ± 2.3 measurements.

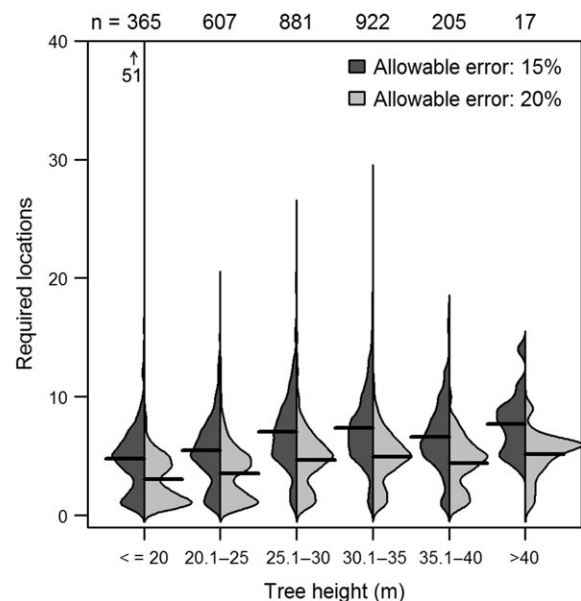


Figure 2 Density distribution of required number of measurement locations per tree grouped by tree height classes. The number of trees per class is shown at the top. The mean per group is indicated by bold horizontal lines.

Fit statistics of the linear model (1) revealed larger errors and higher variances for the smaller dataset 2 (Table 2). A strong autocorrelation of bark thickness within plots and within trees could be shown with the between-tree ($\hat{\sigma}_{\text{tree}}^2$) and within-tree ($\hat{\sigma}_e^2$) variation each making up about one-third of total variation ($\hat{\sigma}_{\text{total}}^2$). The fewer trees were used for model fitting of equation (1), the less runs resulted in equation fits leading to equivalent predictions as the equation fit on the full data (Figure 3). In each sample size simulation with the same number of plots, the number of trees hardly affected the precision of the equation coefficients as long as five or more trees were sampled per plot. Predictions were equivalent to predictions of the full model in more than 94 per

cent of the iterations, as long as five trees were sampled in each of 35 plots.

Model evaluation

Testing the different model forms from the literature showed that equation (6) could best describe both datasets in terms of MAE, RMSE and PB (Table 3). Blocked cross-validation revealed that equations (5) and (9) were best for predictions for datasets 1 and 2, respectively. Large errors in the validation process showed that equation (7) was not useful for predicting double bark thickness of Silver fir. Parameter estimates of all tested models can be found in Table 4. The prediction bias (expressed by the residuals) of equation (5) was equivalent for the two datasets ($P < 0.05$, region of equivalence: 0.5 mm), showing that the model could describe both datasets equally well. However, bark thickness was larger in dataset 1. Predictions for the newly assessed data (dataset 2) were significantly higher ($P < 0.05$, region of equivalence: 0.5 mm) when the model had been fit on dataset 1 compared with predictions made with a model fit from dataset 2.

Table 2 Parameter estimates with their standard error in parentheses, summary of fit statistics, and correlation structure of both datasets using equation (1)

	Parameter	Dataset 1 (1970s)	Dataset 2 (2010s)
Fitting	β_0	4.06 (0.055)	5.39 (0.222)
	β_1	0.05 (0.0002)	0.05 (0.0006)
	MAE (mm)	2.85	3.30
	RMSE (mm)	3.72	4.25
	PB (%)	15.42	16.01
Validation (10-fold CV)	MAE (mm)	2.79	3.37
	RMSE (mm)	3.54	4.24
	PB (%)	15.72	16.36
	$\hat{\sigma}_{\text{total}}^2$	12.88	17.93
	$\hat{\sigma}_{\text{plot}}^2/\hat{\sigma}_{\text{total}}^2$ (%)	33.01	31.40
	$\hat{\sigma}_{\text{tree}}^2/\hat{\sigma}_{\text{total}}^2$ (%)	27.43	33.31
	$\hat{\sigma}_e^2/\hat{\sigma}_{\text{total}}^2$ (%)	39.56	35.29

Note: MAE is the mean absolute error, RMSE is the root mean square error, PB is the percentage bias, $\hat{\sigma}_{\text{total}}^2$ is the total residual variance, $\hat{\sigma}_{\text{plot}}^2$ is the variance between plots, $\hat{\sigma}_{\text{tree}}^2$ is the variance between trees, and $\hat{\sigma}_e^2$ is the variance within trees.

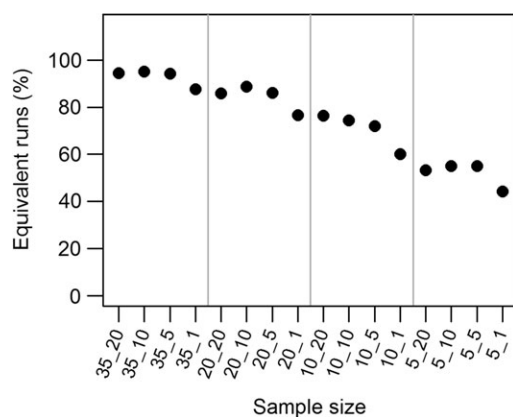


Figure 3 Results from the equivalence test of equation (1) for the 16 different sample size combinations. Those shown are proportion of runs (out of 1000) that resulted in equivalent ($P < 0.05$, region of equivalence = 1 mm) predictions from the full training dataset (larger values are better). Labels on the x-axis refer to the number of plots and the number of trees per plot, respectively.

Geospatial effects

The GAMM fitting of equation (10) and its sub-models obtained the best fit (BIC = 12,109.4) when only the tree-inherent variables d_{ob} , dbh_{ob} , h/H and Age were included as predictors, which is why only these predictors were kept for further analysis. Removing h/H as well only slightly reduced fit quality (BIC = 12,119.1). Total tree height, geographic coordinates, elevation and the climatic factors of temperature and precipitation could not significantly contribute to explaining the variation of bark thickness. The estimated partial effects of the remaining predictors on double bark thickness are shown with their smooth curves in Figure 4. Double bark thickness showed an increasing trend with increasing d_{ob} (Figure 4a) and with dbh_{ob} (Figure 4b). The effect of h/H was small, but obvious at the lowest measurement point at breast height (Figure 4c). A significant increase of DBT with increasing age was observed (Figure 4d).

Bark thickness equations for roundwood volume determination

Relative bark thickness was quite constant in the lower three quarters of tree height and increased slightly further up in the tree. Depending on the bucking pattern with varying log length, midpoints of logs are positioned at different relative tree heights. Nevertheless, results show that bark thickness equations referring to mid-diameters of long logs and of 5-m segments differ only slightly from each other (Table 5, Figure 5). When the equation coefficients for long logs were used as opposed to coefficients for 5-m segments, the calculated log volume was 0.8 and 0.7 per cent higher for long logs ($n = 192$) and 5-m segments ($n = 807$), respectively. The volumes of the individual logs were equivalent (paired TOST, $P < 0.05$, region of equivalence 0.01 m^3) using the two sets of coefficients for the 5-m segments, but not equivalent for the long logs. Predicted bark thickness for small ends of 5-m segments with a diameter of up to 40 cm was higher than for the other tested reference diameters (Figure 5).

Table 3 Summary of fit statistics for mixed-effects models (4)–(9) and the respective prediction bias using only the fixed terms

Dataset			Equation (4) LME	Equation (5) LME	Equation (6) LME	Equation (7) LME	Equation (8) LME	Equation (9) NLME
1 (1970 s)	Fitting	MAE (mm)	1.90	1.90	1.29	1.77	1.89	1.48
		RMSE (mm)	2.64	2.62	1.77	2.48	2.61	2.01
		PB (%)	9.98	9.93	6.92	9.27	9.95	7.90
	Validation (10-fold CV)	MAE (mm)	3.00	2.51	3.47	4.74	3.00	2.60
		RMSE (mm)	3.77	3.23	4.49	5.63	3.76	3.38
		PB (%)	16.88	13.90	19.78	35.88	16.79	14.64
2 (2010 s)	Fitting	MAE (mm)	2.12	2.29	1.50	2.08	1.98	1.83
		RMSE (mm)	2.87	3.06	1.98	2.80	2.64	2.36
		PB (%)	10.05	10.94	7.42	9.85	9.36	9.04
	Validation (10-fold CV)	MAE (mm)	3.58	3.03	3.86	7.38	3.62	3.01
		RMSE (mm)	4.38	3.83	4.74	8.15	4.44	3.77
		PB (%)	17.48	14.76	18.99	60.21	17.51	14.91

Best values per dataset are printed in bold.

Note: LME = linear mixed-effects model; NLME = nonlinear mixed-effects model; MAE = mean absolute error; RMSE = root mean square error; PB, percentage bias.

Bark thickness at midpoints of long logs was smaller for virtual logs of dataset 2 than those of dataset 1 (Table 5, Figure 6). In Germany, the currently suggested boundaries for the diameter classes referring to bark deduction values of 2, 3 and 4 cm have been developed using dataset 1 and are 23, 39 and 56 cm, respectively (Anonymous, 2015). The developed bark thickness equation from dataset 2 would lead to new class boundaries at 25, 43 and 61 cm. Assuming that harvesting intensity and assortments are similar to those of the last 5 years, the application of the new class boundaries would mean a 0.8 per cent annual increase in sales volume of manually measured Silver fir logs.

Discussion

The development of accurate bark thickness equations depends on how well within-tree and between-tree variability is captured by an appropriate sampling design, adequate sample sizes, and the predictor variables in the chosen model.

This study showed that the sampling design with 2-m increments between the sampling locations starting from 1.3 m above ground was adequate to assess the variability within a stem in most cases. Thus, this design is recommended for further studies of species with similar bark morphology. The sampling design with regularly spaced sampling locations (every 2 m along the tree bole) will lead to more measurements with increasing tree height. In contrast, a fixed number of relative positions along the tree bole, as is suggested in other studies (e.g. Korell, 1972; Feduccia and Mann, 1976; Kozak and Yang, 1981; Gordon, 1983; Laasasenaho *et al.*, 2005), could reduce the support for each data point and would require additional effort in the field to recompute relative positions for each single tree. As shown by Stängle *et al.* (2016b), the type of bark gauge used in this study overestimated bark thickness by 0.52 mm (SD = 1.59 mm) when compared with CT-derived measurements on

Norway spruce logs. This could potentially lead to biased bark thickness predictions for Silver fir as well. If a more accurate measurement device was used, a smaller number of measurements per location would be sufficient to reach the same level of accuracy.

The observed high intraclass correlation within the residual variance of equation (1) shows that the relationship between diameter and bark thickness varies strongly between individual trees and plots. This was also supported by the Monte-Carlo simulations representing different sample sizes. The number of sampled plots strongly determined the number of iterations with predicted bark thickness values that were equivalent to predictions from the full-data model (Figure 3). Model choice is an important factor influencing the response in predictive capacity of bark thickness equations on sample size. For Norway spruce it was shown that a more complex model with a higher fit quality required fewer sampled plots and trees per plot (Stängle *et al.*, 2016b). The chosen method of drawing plots and trees with replacement from the training data was necessary to account for differing numbers of trees per sampled plot and differing diameter distributions between plots. The variation observed cannot, therefore, be attributed only to sample size but also to the data-derived uncertainty. For that reason, the suggested sample sizes might slightly overestimate those actually required to achieve the intended precision of predictions.

Results showed that a rather complex nonlinear model (equation (9)) and quite a simple model (equation (5)) were most suited for predicting bark thickness of Silver fir. Model (6) described both datasets best, but it failed to predict bark thickness as precisely as models (5) and (9). The same results were reported for Norway spruce in the same region (Stängle *et al.*, 2017). Depending on the purpose of application, different explanatory variables are available and different degrees of computational complexity are preferable. If bark thickness is to be modelled for inventory data, for example, a complex model with many explanatory variables, such as model (9) is suggested

Table 4 Parameter estimates with their standard error in parentheses of equations (4–9) for both datasets

Dataset	Parameter	Equation (4) LME	Equation (5) LME	Equation (6) LME	Equation (7) LME	Equation (8) LME	Equation (9) NLME
1 (1970s)	β_0	7.198 (0.3779)	4.317 (0.2939)	−0.419 (0.0470)		7.199 (0.3977)	−3.980 (0.0415)
	β_1	0.040 (2.5E−04)	0.009 (3.6E−04)	−4.69E−04 (3.1E−05)	0.055 (0.0012)	0.037 (0.0016)	0.674 (0.0127)
	β_2		0.038 (2.5E−04)	0.606 (0.0084)	−0.010 (8.41E−04)	2.4E−05 (5.03E−06)	5.057 (0.0936)
	β_3			0.060 (0.0015)	0.034 (0.0012)	−3.8E−08 (4.68E−09)	1.423 (0.0224)
	β_4				−2.55E−04 (3.4E−05)		0.016 (3.3E−04)
	β_5						9.2E−04 (3.3E−04)
	β_6						0.254 (0.0195)
	δ	0.692	0.698	−0.098	0.713	0.671	−0.091
	CAR 1	0.914	0.910	0.857	0.903	0.913	0.865
	σ	0.053	0.050	0.183	0.044	0.059	0.192
2 (2010s)	β_0	8.641 (0.4824)	2.403 (0.5834)	−0.390 (0.1343)		8.410 (0.6684)	−5.261 (0.1841)
	β_1	0.035 (7.1E−04)	0.013 (0.0010)	−3.7E−04 (8.4E−05)	0.032 (0.0045)	0.034 (0.0047)	1.280 (0.0832)
	β_2		0.034 (7.1E−04)	0.606 (0.0274)	−0.003 (0.0030)	1.9E−05 (1.3E−05)	3.021 (0.1699)
	β_3			0.039 (0.0063)	0.013 (0.0042)	−2.8E−08 (9.6E−09)	2.270 (0.1070)
	β_4				1.6E−04 (1.3E−04)		0.016 (9.1E−04)
	β_5						0.007 (0.0011)
	β_6						0.511 (0.1260)
	δ	0.520	0.538	−0.277	0.526	0.478	−0.308
	CAR 1	0.898	0.897	0.824	0.891	0.886	0.839
	σ	0.148	0.132	0.548	0.140	0.178	0.751

Note: LME = linear mixed-effects model; NLME = nonlinear mixed-effects model; δ = random effect variance; CAR1 = first-order continuous autoregressive term; σ = residual standard error.

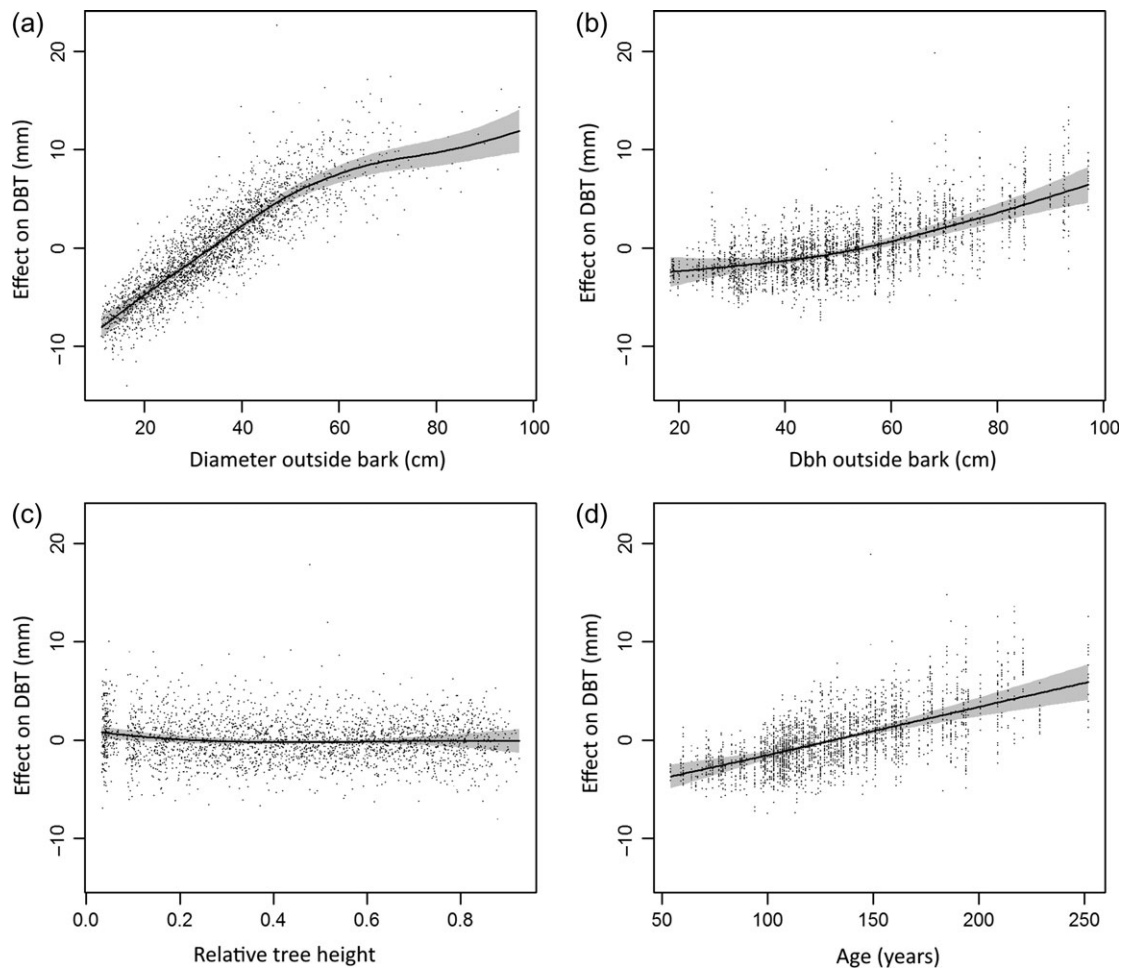


Figure 4 Estimated smooth functions describing the partial effect of the four covariates on double bark thickness (DBT): (a) the smooth of diameter outside bark, (b) the smooth of dbh outside bark, (c) the smooth of relative tree height, and (d) the smooth of age. Shaded areas are 95% confidence intervals.

Table 5 Best fit model coefficients with their standard error in parentheses and coefficient of determination of the functions (1) or (11) predicting double bark thickness with different reference diameters for datasets 1 and 2. All sampling locations means every 2 m in each tree.

Dataset	Reference diameter	Equation	β_0	β_1	β_2	R^2
1	long logs: midpoint	(11)	-0.836 (0.5118)	0.756 (0.0341)	-0.002 (0.0005)	0.79
2	all sampling locations	(1)	5.322 (0.2330)	0.451 (0.0064)	-	0.68
2	long logs: midpoint	(1)	0.946 (0.8830)	0.563 (0.0241)	-	0.74
2	5-m segments midpoint	(1)	3.848 (0.4656)	0.498 (0.0123)	-	0.67
2	5-m segments small end	(1)	5.691 (0.3365)	0.463 (0.0104)	-	0.71

because taper curves outside bark are often available. If, in contrast, total and relative tree height are not known, as in the situation when a harvester measures a tree, the best possible model without these covariates should be selected (equation (5)). Effects of total tree height were generally absent and relative tree height only marginally explained variation in bark thickness. Equation (7), which is mainly based on relative and total tree height as predictors, cannot be recommended for Silver fir, although it was proposed for different *Pinus* species (Cao and

Pepper, 1986). The high influence of absolute and relative tree height on bark thickness variation for pines was also confirmed by Murphy and Cown (2015), who reported bark volume variation within trees, between trees, and between sites for *Pinus radiata* (D. Don). For Norway spruce, effects of tree height and relative tree height on bark thickness variation were shown as well (Stängle et al., 2017). As Norway spruce and most *Pinus* species are less shade tolerant than Silver fir, a higher correlation between total tree height and age can be expected for

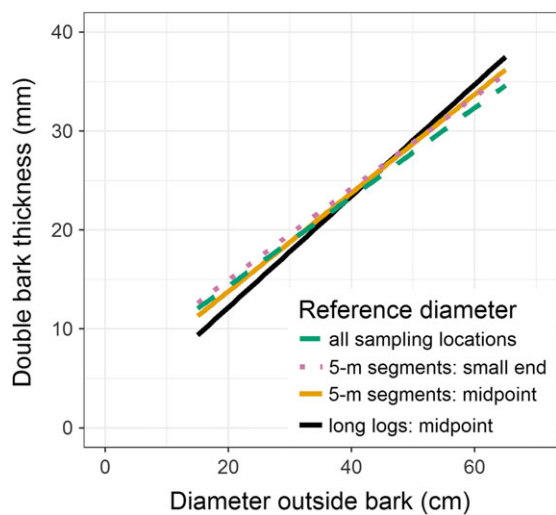


Figure 5 Regression lines describing double bark thickness against the rounded diameter using different reference diameters, which were gained from different bucking variants of virtual logs produced from dataset 2. For comparison, also all sampled locations (every 2 m along the tree bole) are used as reference diameters.

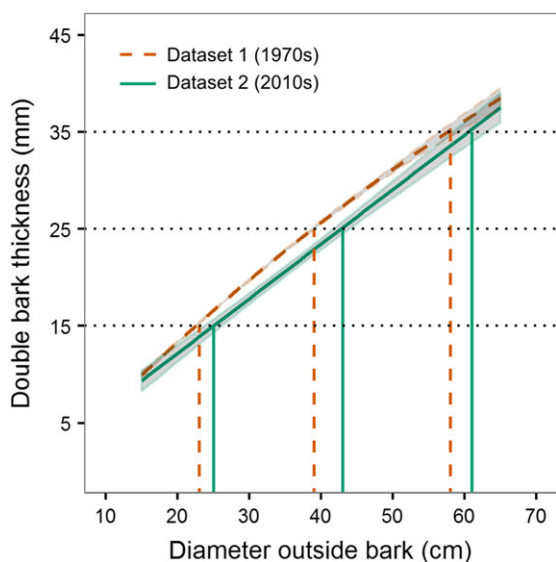


Figure 6 Double bark thickness plotted against the rounded mid-diameter with 95% confidence intervals of the regression lines. Vertical lines indicate the class boundaries of integer bark deduction values (horizontal lines: class midpoints 10–20, 30–40 and 40–50 mm dbt). Shifts are resulting from more shallow-barked Silver firs in the more recent data (1970s vs 2010s).

those species. Therefore, the effect of total tree height for those species could, at least partially, be caused by collinearity.

The variation observed between the sample plots could not be linked to large-scale spatial patterns, such as climatic zones and effects of geographic coordinates were generally absent. Moreover, the climatic factors of temperature and precipitation as well as elevation could not explain differences between plots. Tree age, however, was found to have a significant positive

effect on bark thickness. These findings are in accordance to those of [Laasasenaho et al. \(2005\)](#), [Sonmez et al. \(2007\)](#) and [Stängle et al. \(2017\)](#) for Norway spruce. This leads to the conclusion that higher growth rates result in smaller relative bark thickness for those species. Other studies also show that increased site quality, which can usually be linked to faster growth of individual trees, reduced bark thickness of Silver fir ([Božić et al., 2007](#)). It seems that no single environmental predictor can explain differences in bark thickness. Rather, the combination of many factors influencing the productivity of a stand and the growth of individual trees is required. For future studies of bark thickness, the assessment of tree age or site index is, therefore, strongly recommended. The observation that faster growing trees have thinner bark might explain the observed thinner bark in the more recent measurements of dataset 2. Since growth of Silver fir in southwestern Germany has slightly accelerated in the twentieth century ([Kohnle et al., 2014](#)), it can be expected that the trees of the new dataset have grown faster than those trees measured 40 years earlier.

The small difference in relative bark thickness along the stem was apparent when bark thickness equations were fitted on mid-diameters of long logs and of 5-m segments. As there were hardly any differences in volume calculation, the same equation coefficients could be used for different log assortments. However, the small ends of 5-m segments showed larger relative bark thickness as they are positioned further up in the tree. This difference, especially for small diameters, suggests that different equation coefficients should be used for bucking optimization that is based on minimum small end diameters. Alternatively, a model, which includes information on the relative height of the sampling location, could be chosen.

Thinner bark in the more recent measurements resulted in different diameter classes for integer bark deduction for the two datasets. The values that are currently in use in Germany ([Anonymous, 2015](#)) are based on the older measurements. Therefore, they tend to overestimate bark thickness and lead to biased inside-bark volume estimations.

Conclusion

To parameterize bark thickness equations, within-tree and between-tree variation in Silver fir bark thickness requires several sampling locations per tree and a minimum of five sampled trees in at least 35 plots in the study region. Bark thickness of Silver fir can be described well using tree diameter at the sampling location and the breast height diameter as predictor variables. Faster growing trees were shown to have relatively thinner bark. From the demonstrated influence of tree age – and therefore growth rate – on bark thickness, it can be inferred that changing growth conditions can alter bark allometry, which is why bark thickness equations should be regularly reviewed for their validity.

For measuring roundwood, the same bark deduction values can be suggested for long logs (6–21 m) and 5-m segments. For bucking optimization based on minimum small end diameters, different equation coefficients that reflect higher relative bark thickness at larger relative tree height are recommended. The findings show the need to update the current bark deduction values for manual log measurement in the state of

Baden-Württemberg in order to estimate log volume more accurately. As the same values are currently used in large parts of Central Europe, further validation measurements in other federal states in Germany and in neighbouring countries are suggested.

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Conflict of interest statement

None declared.

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